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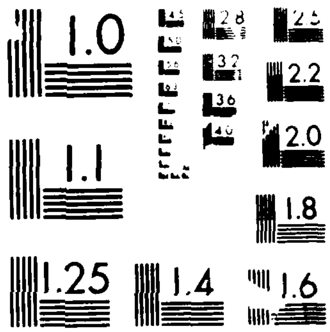

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Technical Report No. 2

UNSTABLE ELASTIC MATERIALS AND THE VISCO-ELASTIC RESPONSE OF BARS IN TENSION

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Technical Report No. 2

UNSTABLE ELASTIC MATERIALS AND THE VISCO-ELASTIC RESPONSE OF BARS IN TENSION

by

Rohan Abeyaratne* and James K. Knowles**

*Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

**Division of Engineering and Applied Science
California Institute of Technology
Pasadena, California 91125

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Some homogeneous elastic materials are capable of sustaining finite equilibrium deformations with discontinuous strains. For materials of this kind, the energetics of isothermal, quasi-static motions may differ from those conventionally associated with elastic behavior. When equilibrium states involving strain jumps occur during such motions, the rate of increase of stored energy in a portion of the body may no longer coincide with the rate of work of the external forces present. In general, energy balance now includes an additional effect due to the presence of moving strain discontinuities. As a consequence, the macroscopic response of the body may be dissipative. This fact makes it possible to model certain types of inelastic behavior in solids with the help of such "unstable" elastic materials; see, for example, Abeyaratne and Knowles (1987a,b,c).

The purpose of the present note is to illustrate behavior of this kind with the help of an especially simple example involving the extensional deformations of a bar treated as a one-dimensional continuum. The bar is composed of an unstable elastic material of the type considered by Ericksen (1975). We show that the force-elongation relation (or "macroscopic response") of the bar during a quasi-static motion may be viscoelastic when a moving strain jump is present, even though the underlying constitutive law is elastic in the sense that, at each particle, the present stress is determined by the present strain.

Consider an elastic bar which, in the reference configuration, occupies the interval $[0,L]$ of the x-axis and has constant cross-sectional area A. In a deformation, the particle at x is carried to $y = x + u(x)$, where u is the displacement. We assume that u is continuous on $[0,L]$, and that $u'(x)$ exists and is continuous for $0 \leq x \leq s$ and $s \leq x \leq L$, but $u'(s+)$ and $u'(s-)$ may differ; here $0 < s < L$. If $x \neq s$, the strain at x is $\varepsilon(x) = u'(x)$. It is required that $\varepsilon(x) >$

-1 for $x \neq s$, so that the mapping $x \to y$ is invertible. We take the left end of the bar to be fixed, so that $u(0) = 0$.

Suppose that the material is elastic with strain energy per unit reference volume $W(\varepsilon)$. The nominal stress response function is then

$$\hat{\sigma}(\varepsilon) = W'(\varepsilon), \tag{1}$$

so that the nominal stress in the bar at particle x is $\sigma(x) = \hat{\sigma}(\varepsilon(x))$. We shall be concerned with the special "trilinear" stress response function given by

$$\hat{\sigma}(\varepsilon) = \begin{cases} \mu\varepsilon \quad , & -1 < \varepsilon \leq \varepsilon_M \, , \\ \dfrac{\mu\varepsilon_M}{\varepsilon_m - \varepsilon_M}(\varepsilon_m + \varepsilon_M - 2\varepsilon) \, , & \varepsilon_M \leq \varepsilon \leq \varepsilon_m \, , \\ \mu(\varepsilon - \varepsilon_o), & \varepsilon_m \leq \varepsilon < \infty \, . \end{cases} \tag{2}$$

The graph of $\hat{\sigma}(\varepsilon)$ is shown in Figure 1, where the meanings of the constants μ, ε_M, ε_m and $\varepsilon_o = \varepsilon_M + \varepsilon_m$ are made clear. One may think of the material as exhibiting three phases: the first phase is represented by the rising branch of the stress-strain curve through the origin, the third phase corresponds to the final rising branch, and the declining portion of the curve represents an unstable intervening second phase. In the present case, the first and third phases are both associated with the same modulus μ.

In the absence of body force, the bar will be in equilibrium if the nominal stress $\sigma(x)$ satisfies

$$\sigma(x) = \sigma = \text{constant} \ , \ 0 \leq x \leq L \ . \tag{3}$$

A deformation with a strain discontinuity of the kind described above will correspond to an equilibrium state if it is of the form

$$u(x) = \begin{cases} \varepsilon_1 x , & 0 \leqq x \leqq s, \\ \varepsilon_2 x + (\varepsilon_1 - \varepsilon_2)s , & s \leqq x \leqq L, \end{cases} \tag{4}$$

where the strains ε_1 and ε_2 are constant and such that

$$\hat{\sigma}(\varepsilon_1) = \hat{\sigma}(\varepsilon_2) = \sigma . \tag{5}$$

We shall be concerned here only with those deformations of the form (4) in which

$$-\varepsilon_M \leqq \varepsilon_1 \leqq \varepsilon_M , \quad \varepsilon_m \leqq \varepsilon_2 \leqq \varepsilon_M + \varepsilon_0 , \tag{6}$$

so that only phases one and three are represented. For an equilibrium state of this kind, the total energy stored in the bar is

$$E = AW(\varepsilon_1)s + AW(\varepsilon_2)(L - s) . \tag{7}$$

Now suppose that the bar is in a quasi-static motion during which, at each instant t, the displacement u(x,t) is of the form (4), with $s = s(t)$, $\varepsilon_1 = \varepsilon_1(t)$ and $\varepsilon_2 = \varepsilon_2(t)$. The restrictions (5) and (6) are to hold for all t, with $\sigma = \sigma(t)$. Assume that s(t), $\varepsilon_1(t)$ and $\varepsilon_2(t)$ are continuous and piecewise continuously differentiable in t. During such a motion, the energy E(t) is given by (7) with ε_1, ε_2, and s now functions of time. A direct calculation gives

$$\dot{E}(t) = F(t)\dot{\delta}(t) + A[-f(t)]\dot{s}(t), \tag{8}$$

where $F(t) = \sigma(t)A$ is the force,

$$\delta(t) = u(L,t) = \varepsilon_1(t)s(t) + \varepsilon_2(t)(L - s(t)) \tag{9}$$

is the elongation of the bar at time t, and f(t) is given by

$$f(t) = W(\varepsilon_2(t)) - W(\varepsilon_1(t)) - \sigma(t)(\varepsilon_2(t) - \varepsilon_1(t)). \quad (10)$$

The first term on the right in (8) is the rate of work of the force acting on the end $x = L$ of the bar. The second term may be interpreted as the rate of work done by a fictitious "traction" $f(t)$ in moving the strain discontinuity at $x = s(t)$. For the stress-strain relation (2), one finds using (1),(6) and (10) that

$$f(t) = -\varepsilon_0\sigma(t). \quad (11)$$

If the motion takes place isothermally, the second law of thermodynamics requires that $\dot{E} - F\dot{\delta}$ be nonnegative; thus the motion must be such that

$$f(t)\dot{s}(t) \geq 0. \quad (12)$$

Because of (5) and (6), specifying the history of the stress acting on the bar determines $\varepsilon_1(t)$ and $\varepsilon_2(t)$, but leaves undetermined the location $s(t)$ of the strain jump, and therefore the elongation history $\delta(t)$ of (9) as well.

In formulas (7) and (9), one may regard s as an "internal variable" akin to those arising in microstructural theories of plasticity; see, for example Rice (1971) . This suggests that, by analogy with such theories, the constitutive description of the material should be augmented by relating the fictitious traction $f(t)$ to the velocity of the moving discontinuity $\dot{s}(t)$. One form that such a "kinetic relation" might take is

$$\dot{s}(t) = V(f(t)) , \quad (13)$$

where V is a function determined by the material. Using (11), one infers from (13) that

$$\dot{s}(t) = V(-\varepsilon_0 \sigma(t)) \ . \tag{14}$$

Note from (12) that every admissible V must be such that $V(f)f \geq 0$.

Let $\gamma(t) = \delta(t)/L$ be the *relative* elongation of the bar. The relation between γ and F - the macroscopic response of the bar - may now be determined as follows. Using (2) and (6) in (9) yields

$$\delta(t) = \varepsilon_0[L-s(t)] + \sigma(t)L/\mu \ , \tag{15}$$

from which, with the help of (14), one finds the nonlinear viscoelastic relation

$$\dot{\gamma}(t) = \dot{F}(t)/\mu A - \varepsilon_0 V(-\varepsilon_0 F(t)/A)/L \ . \tag{16}$$

If in particular the kinetic response function V is specified through $V(f) = f/\nu$, where ν is a positive constant, (13) provides a linearly "viscous" kinetic relation between f and $\dot{s}$. The macroscopic response relation (16) specializes to

$$\dot{\gamma}(t) = \dot{F}(t)/\mu A + \varepsilon_0^2 F(t)/\nu LA \ . \tag{17}$$

This is precisely the form of the response relation characteristic of the so-called "Maxwell" spring-dashpot model of elementary viscoelasticity. Other forms of macroscopic response can be obtained by replacing (13) by a more general kinetic relation.

A more extensive discussion of the one-dimensional theory of quasi-static motions of bars composed of unstable elastic materials may be found in Abeyaratne and Knowles (1987c).

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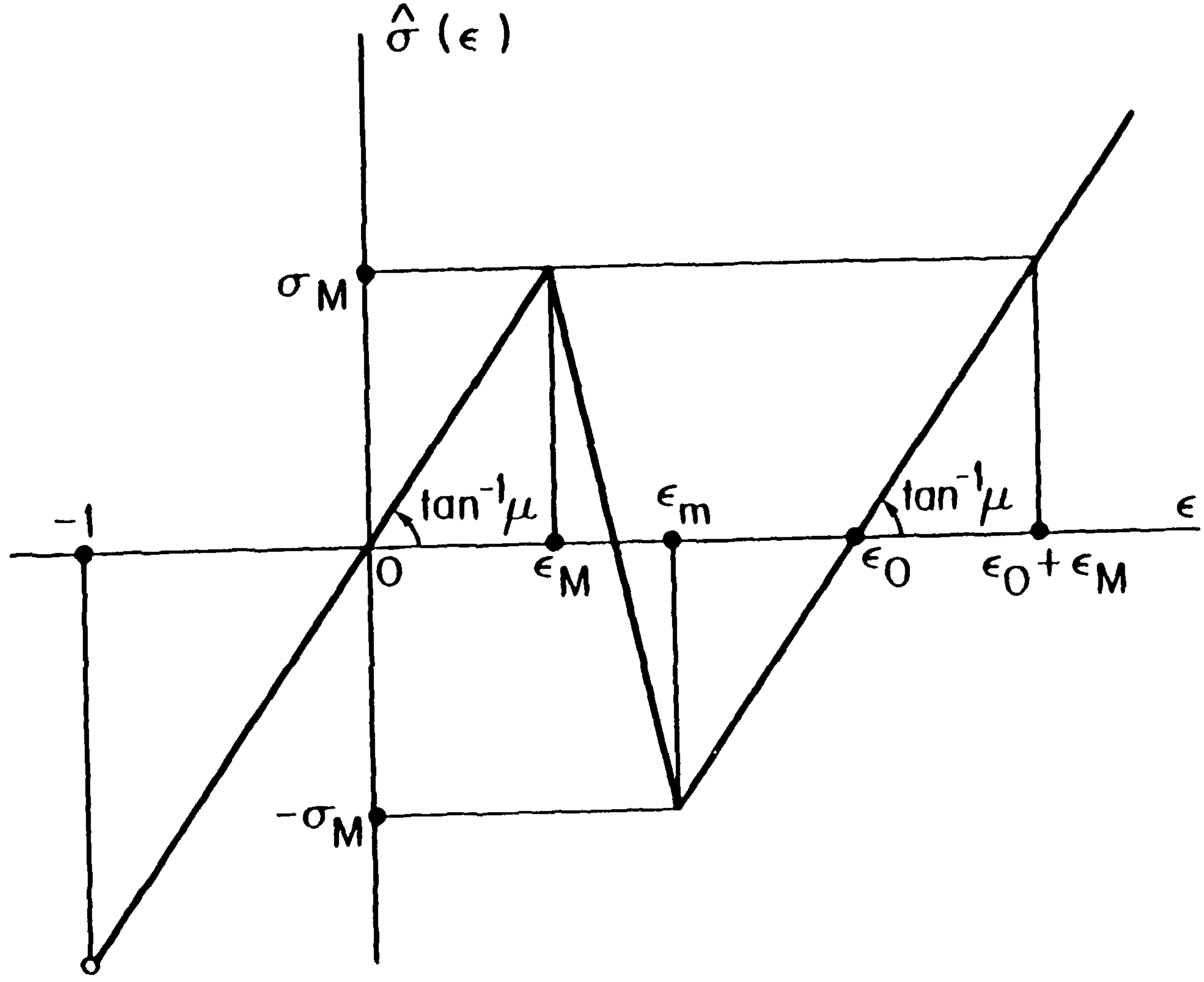

FIGURE 1. STRESS RESPONSE FUNCTION.

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The force-elongation relation (or macroscopic response) of a bar of unstable elastic material is in general dissipative. In this note, it is shown that the macroscopic response may be of conventional viscoelastic type.

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